

On the construction of a Fractal Spectrum for Data Analysis

Roberto N. Padua¹

Daisy R. Palompon²

Efren O. Barabat³

Mark S. Borres³

Randy K. Salazar³

Negros Oriental State University, Philippines¹

Cebu Normal University, Philippines²

University of San Jose – Recoletos, Philippine³

mark_borres@yahoo.com

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ABSTRACT

Multifractal probability distributions are defined as mixture of n monofractal distributions. The exponents λ_i are the fractal dimensions which determine the information- filling property of an observation x_i from this distribution. A useful device for examining multifractal observations is the multifractal spectrum. The fractal spectrum was shown to be a one-to-one function, monotonically increasing with x on a logarithmic scale for non-fractal distributions. Similarly, multifractal distributions viewed on a larger scale (s) have a spectrum that behaves like power function As^{-k} but when viewed on a smaller scale, it behaves like a concave downward quadratic function $A(s - s_0)^2 + B(s - s_0) + C$ where A , B and C are parameters to be estimated.

Keywords: *fractal distribution, fractal, fractal spectrum, multifractal, power law*

I. INTRODUCTION

The utility of multifractal analysis in the analysis of seismic data in Italy was demonstrated by Lapenna, Macchiato and Telesca (1998), in the Philippines by Panduyos, Villanueva and Padua (2014), and in other countries by various authors. Of these multifractal models of seismic data, the main tool used was Legendre's multifractal spectrum which essentially involves finding a

sequence of multifractal manifolds which can be expressed in terms of power laws. In Padua and Barabat & Regolado (2013) a simpler multifractal spectrum $\lambda(s)$ was found useful in fractal data analysis.

Multifractal probability distributions are define as mixture of m monofractal distributions in the sense of Tukey (1972):

$$(1) \quad f(x) = w_1 f_1(x) + w_2 f_2(x) + \cdots + w_m f_m(x) \quad , x \geq \theta$$

where:

$$(2) \quad f_i(x) = \left(\frac{\lambda_i - 1}{\theta}\right) \left(\frac{x}{\theta}\right)^{-\lambda_i} \quad , i = 1, 2, 3, 4, \dots, m \quad \sum_{i=1}^m w_i = 1 \quad , \quad w_i \geq 0.$$

The exponents λ_i are the fractal dimensions which determine the information-filling property of an observation x_i from this distribution. In several papers, authors claimed that fractal observations are, in fact, more pervasive in real-life than normal observations Selvam (2009); Lapenna, Macchiato, and Telesca (1998); Padua, Borres and Salazar (2013). As such, fractal distributions need to be examined more closely and classical normal-based methods, reviewed.

A useful device for examining multifractal observations is the multifractal spectrum. The current multifractal spectrum in use is the Legendre' spectrum but its application is largely confined to scientists in specialized fields because of its complexity. Padua, Barabat, Borres, and Salazar (2013) suggested a simpler version of a multifractal spectrum, namely:

$$(3) \quad \lambda(s) = 1 - \frac{\log(1-\alpha)}{\log(\frac{x}{\theta})}, \quad \theta \geq x, \quad \alpha = F(x)$$

$$\lambda(s) = 1 - s \log(1 - F(x)), \quad s = \frac{1}{\log(\frac{x}{\theta})}.$$

that behaves in exactly the same way as the Legendre's spectrum. Thus, monofractal $\lambda(s)$ -spectrum is a cluster of points or a single point while multifractal spectra are single-humped, continuous functions of scale s .

We present some results in relation to the behavior of $\lambda(s)$ in this paper. These results describe the behavior of the $\lambda(s)$ -spectrum when applied to various probability distributions, including the multi-fractal distributions.

2.0 The $\lambda(s)$ - spectrum

Let $x_1, x_2, \dots, x_n$ be iid $G(\cdot)$ where $G(\cdot)$ is an unknown absolutely continuous distribution with respect to a Lebesgue measure. Suppose also that $x_i \geq \theta$ for each i and $\theta > 0$. The idea is to fit a fractal distribution:

$$(4) \quad F(x) = 1 - \left(\frac{x}{\theta}\right)^{1-\lambda}, \quad \lambda > 0, \quad x \geq \theta$$

to the quantiles $x_{\alpha_1} \leq x_{\alpha_2} \leq \dots \leq x_{\alpha_n}$ such that $G(x_{\alpha_n}) = \alpha_n$.

Thus,

$$(5) \quad G(x_\alpha) = \alpha = F(x_\alpha) = 1 - \left(\frac{x}{\theta}\right)^{1-\lambda}$$

which reduce to:

$$(6) \quad (1-\lambda) \log\left(\frac{x}{\theta}\right) = \log(1-\lambda).$$

we obtain:

$$(7) \quad \lambda = 1 - \frac{\log(1-\alpha)}{\log(\frac{x}{\theta})}$$

From which the fractal spectral function:

$$(8) \quad \lambda(s) = 1 - \log(1-\alpha) s, \quad s = \frac{1}{\log(\frac{x}{\theta})}$$

is obtained.

The fractal spectrum (7) was shown to be a one-to-one function, monotonically increasing with x on a logarithmic scale for non-fractal distributions. Instead of examining the observations on the data space, we propose to examine them in the spectral space. The value of θ used in (8) serves as a powerful "microscope" that enhances the detailed picture of the spectrum $\lambda(s)$ in terms of its finer structures.

We note that if x comes from fractal distribution with fractal dimension λ , then $s = \frac{1}{\log(\frac{x}{\theta})}$ decreases with increasing x and decreasing θ . For a fixed observation x , we can increase (decrease) by decreasing θ (increasing θ), so that the value of θ serves to sharper the focus on the features of a fractal set. Viewed on a large scale, monofractal distributions have singular spectra ($P(\lambda(x) = \lambda_0) = 1$) but when viewed on a lower scale, the spectral function forms a horizontal line (slope = 0). Similarly, multifractal distributions viewed on a larger scale (s) have a spectrum that behaves like power function $A s^{-k}$ but when viewed on a smaller scale, it behaves like a concave downward quadratic function $A(s - s_0)^2 + B(s - s_0) + C$ where A , B and C are parameters to be estimated.

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